

Separable	$f(x) = g(y) \frac{dy}{dx}$	Separate $\frac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x, y) + N(x, y) \cdot y' = 0$	Test: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Find f where $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$ Solution: $f(x, y) = c$
Constant coefficient	$a_2 y'' + a_1 y' + a_0 y = 0$	Two real roots r_1, r_2 use $e^{r_1 x}$ and $e^{r_2 x}$ If double real root r use e^{rx} and $x e^{rx}$ If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x} \cos(\beta x)$ and $e^{\alpha x} \sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$ y_1 and y_2 sols. to homogeneous eqn.	$y_p = v_1 y_1 + v_2 y_2$ $v_1 = \int \frac{-y_2 \cdot b(x)}{W(y_1, y_2)} dx \quad v_2 = \int \frac{y_1 \cdot b(x)}{W(y_1, y_2)} dx$
Reduction of Order	$y'' + a_1(x)y' + a_0(x)y = 0$ on the interval I	y_1 is a solution that isn't zero on I $y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)dx}}{y_1^2} dx$
Euler's method	$y' = f(x, y)$ $y(x_0) = y_0$	$x_n = x_{n-1} + h$ $y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$

$b(x)$	y_p guess for undetermined coefficients
constant	A
degree one polynomial such as: $5x - 3$ or $2x$	$Ax + B$
degree two polynomial such as: $10x^2 - x + 1$ or $x^2 + x$ or $2x^2 - 3$	$Ax^2 + Bx + C$
$\sin(kx)$ where k is a constant	$A \cos(kx) + B \sin(kx)$
$\cos(kx)$ where k is a constant	$A \cos(kx) + B \sin(kx)$
exponential such as: e^{kx} or $-2e^{kx}$	Ae^{kx}
degree one poly times exponential such as: xe^{kx} or $(2x + 1)e^{kx}$	$(Ax + B)e^{kx}$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1' \quad \text{quadratic formula: } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

$$\text{Power series / Taylor series: } f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f''''(x_0)}{4!}(x - x_0)^4 + \dots$$

$$(fg)' = f'g + fg' \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad \int u dv = uv - \int v du \quad \frac{d}{dx} \cos(x) = -\sin(x) \quad \frac{d}{dx} \sin(x) = \cos(x)$$

$$\int \sin(x) dx = -\cos(x) \quad \int \cos(x) dx = \sin(x) \quad \int \frac{dx}{1+x^2} = \tan^{-1}(x) \quad \int \tan(x) dx = \ln|\sec(x)|$$